

# HALL EFFECTS ON HYDROMAGNETIC NON-NEWTONIAN CONVECTIVE FLOW IN A ROTATING CHANNEL WITH MASS TRANSFER

S.S.S. Mishra<sup>1</sup>, G.S. Ray<sup>2</sup>, S. Biswal<sup>3</sup> and M. Jena<sup>4</sup>

1. Department of Physics, U.G.S. Mahavidyalaya, Sakhigopal, 752012
2. Chairman, CHSE, Odisha, Bhubaneswar, India
3. Former Professor of Physics, SERC, Bhubaneswar, India
4. Department of Physics, College of Basic Sciences and Humanities, Bhubaneswar, India

## Abstract

Hall effects on combined free and forced convective flow of a viscous-elastic incompressible electrically conducting fluid between the horizontal perfectly conducting plates under the action of a uniform transverse magnetic field applied parallel to the axis of rotation is studied. Exact solution of the governing equation is obtained in closed form. It is observed that Hall current exerts stabilizing influence on the primary flow at the upper plate due to shear stress while at the lower plate Grashof number causes separation on the secondary flow. The rate of heat transfer at both the plates are derived. It is found that the Hall current and rotation exert reverse flow of heat at the upper plate when the numerical value is equal to two as the Grashof number is being referred. Mass transfer is analysed by solving the constitutive equations for concentration.

**Keywords :** Hall effect / MHD / Non-Newtonian fluid / Rotating channel / Mass Transfer.

## 1. INTRODUCTION

The theory of rotating fluid is highly important in various technological situations determine the behavior of conducting fluid with low Prandtl number to which the interaction between electromagnetic force to coriolis force is subjected to the action of modify the mechanical behavior of the system. Mazumder et al<sup>1</sup>, Datta and Jana<sup>2</sup> and Seth and Ghosh<sup>3</sup> investigated the combined effects of free and forced convection flow with Hall effects in a non-rotating system neglecting induced magnetic field under different conditions. However, the influence of such fluid flow problem which lie in their application of geophysical and astrophysical interest, is the study of a steady free and forced convection flow with Hall effects in a rotating

system, has not received attention in literature where induced magnetic field is taken into account.

The MHD flow of a conducting liquid between two non conducting parallel plates in the presence of a transverse magnetic field was studied by Hartmann<sup>4</sup>, Agrawal<sup>5</sup> and Soundalgekar<sup>6</sup>. Ghosh<sup>7</sup> has analysed Hall effects on Hydromagnetic non Newtonian convective flow in a rotating channel with mass transfer. Mishra, Biswal and Ray<sup>8</sup> have studied MHD free convection flow of a rotating non Newtonian fluid past an isothermal vertical porous plate with varied species concentration. The problem of Heat and mass transfer in the MHD flow of a visco elastic fluid in a rotating porous channel with radiative heat have been investigated by Jena, Goswami and Biswal<sup>9</sup>. Biswal, Ray and Mishra<sup>10</sup> has studied hall effects an hydromagnetic convective flow of a viscous fluid through a rotating porous channel with heat and mass transfer.

In the present paper, we consider the effect of Hall current on the combined free and forced convection flow of an electrically conducting visco-elastic incompressible fluid between two horizontal perfectly conducting plates rotating with an uniform angular velocity about an axis normal to their planes under the action of a uniform transverse magnetic field applied parallel to the axis of rotation. Exact solution of the governing equations for the fully-developed flow is obtained in closed form. The solution in dimensionless form contains five flow parameters viz.  $M^2$  (the square of the Hartmann number),  $K^2$  (the rotation parameter),  $G$  (Grashof number),  $G^*$  (modified Grashof number) and  $m$  (the Hall parameter). Asymptotic behavior of the solution is analysed for large values of  $K^2$  and  $M^2$ . The shear stress at both the plates due to primary and secondary flows are derived. The rate of heat transfer at both the plates are presented numerically. It is found that there arise flow reversals in the primary as well as secondary flow directions for  $G=0$  and  $G^*=0$ , while Hall current and rotation exert a destabilizing influence on the primary flow whereas the rotation has a stabilizing influence on the secondary flow. Also it is noticed that there is a reverse flow of heat at the upper plate on increasing  $m$ ,  $K^2$  for  $G = 2$ ,  $G^*=0$ .

## 2. FORMULATION OF THE PROBLEM AND ITS SOLUTIONS

Consider the steady fully-developed combined free and forced convection flow of an electrically conducting viscous incompressible fluid between two infinite horizontal perfectly conducting parallel plates  $y = \pm L$  under the influence of a constant pressure gradient acting along x-axis and a uniform transverse magnetic field  $H_0$  applied parallel to y-axis about which both the fluid and plates are in a state of rigid body rotation with uniform angular velocity  $\Omega$ . The plates are cooled or heated by a constant temperature gradient along the x-direction so that the temperature varies linearly along the plate.

Since the plates are infinite along x and z directions all physical quantities except pressure will be the function of y only. It may be easily shown that the following assumptions are compatible with the fundamental equations of magnetohydrodynamics.

$$q = (u', 0, w'), H = (H'_x, H_0, H'_z) \quad (2.1)$$

Under the assumptions (2.1) which correspond to the fundamental equations of magnetohydrodynamics in a rotating frame of reference, the equation of momentum and the Ohm's law for a moving conductor taking Hall current into account.

$$(q \cdot \nabla)q + 2\Omega \times q = \frac{1}{\rho} \nabla p + \nu \nabla^2 q + \frac{K_0}{\rho} (\nabla^3 q) + \frac{\mu_e}{\rho} J \times H + g \{1 - \beta(T - T_0) - B^*(C - C_0)\} \hat{K} \quad (2.2)$$

$$J + \frac{\omega_e \tau_e}{H_0} (J \times H) = \sigma [E + \mu_e q \times H] \quad (2.3)$$

where  $q$ ,  $E$ ,  $J$  and  $H$  are respectively, the velocity vector,  $\rho$ ,  $\nu$ ,  $\mu_e$ ,  $p$ ,  $\sigma$ ,  $\omega_e$ ,  $\tau_e$ ,  $g$ ,  $\beta$ ,  $T$  and  $T_0$  are respectively, the fluid density, kinematic coefficient of viscosity, magnetic permeability, modified pressure including centrifugal force, electrical conductivity, cyclotron frequency, electron collision time, gravity, the coefficient of thermal expansion, the fluid temperature and the temperature in the reference state.  $\hat{K}$  is the unit vector along y-axis.

Assuming uniform axial temperature variation along the channel walls, the fluid temperature may be considered as

$$T - T_0 = Nx + \phi(y) \tag{2.4}$$

under the assumption (2.1), taking y-component on integrating the momentum equation (2.2), reduces to

$$P = -\rho gy + \beta g \int (T - T_0) dy + (C - C_0) dy - \frac{1}{2} (H_x^2 + H_z^2) + F(x) \tag{2.5}$$

Combining eqns. (2.2) and (2.3) with the help of eqn. (2.5) in dimensionless form, we obtain

$$R_c \frac{d^3 F}{d\eta^3} + \frac{d^2 F}{d\eta^2} + R \frac{dF}{d\eta} + M^2 \frac{dh}{d\eta} - G\eta - G^*\eta = -I - 2iK^2 F, \tag{2.6}$$

Integrating (2.6), we get

$$R_c \frac{d^3 F}{d\eta^2} + \frac{dF}{d\eta} + RF + M^2 \frac{dh}{d\eta} - (G + G^*)\eta = -(1 + 2iK^2 F)$$

$$\frac{d^2 h}{d\eta^2} + \frac{1}{(1 + im)} \frac{dF}{d\eta} = 0, \tag{2.7}$$

Where  $\eta = \frac{y}{L}$ ,  $u_1 = u' \frac{L}{\nu} P_x$ ,  $W_1 = W' \frac{L}{\nu} P_x$ ,  $R = \frac{uL}{\nu}$

$$H_x = \frac{H'_x}{\sigma\mu_e} \nu H_0 P_x, \quad H_z = \frac{H'_z}{\sigma\mu_e} \nu H_0 P_x, \quad R_c = \frac{K_0}{\rho P_x V^2} \tag{2.8}$$

$$P_x L^3 = \left( -\frac{dF}{dx} \right) / \rho \nu^2, \quad G = g\beta \frac{NL^4}{\nu^2} P_x \text{ is the Grashof number,}$$

$$G^* = g\beta^* \frac{NL^4}{\nu^2} P_x, \text{ modified Grashof number}$$

$M = \mu H_0 L \left( \frac{\sigma}{\rho \nu} \right)^{1/2}$  is the Hartmann number,  $K^2 = \frac{\Omega L^4}{\nu}$  is the rotation parameter

which is reciprocal of Ekaman number,  $m = \omega_e \tau_e$  is the Hall current parameter,  $F = u_1 + iw_1$  and  $h = H_x + iH_z$ ,  $R$  is the Reynolds number,  $R_c$  is the non-Newtonian parameter.

Equation (2.4) shows that positive or negative values of  $N$  correspond to heating or cooling along the channel walls. Considering  $P_x > 0$ , it follows from the definition of  $G$  and  $G$  is less than or greater than 0 according as the channel walls are heated or cooled in the axial direction.

The boundary conditions for the velocity field are

$$F = 0 \text{ at } \eta = \pm 1 \tag{2.9}$$

Since the plates are perfectly conducting, the boundary conditions for the magnetic field are

$$\frac{dh}{d\eta} = 0 \text{ at } \eta = \pm 1 \tag{2.10}$$

Since the channel is symmetric at  $\eta = 0$ , the boundary condition for the magnetic field at  $\eta = 0$  may be assumed as (see Nanda and Mohanty)<sup>11</sup>

$$h = 0 \text{ at } \eta = 0 \tag{2.11}$$

Equations (2.6) and (2.7) together with the boundary conditions (2.9) to (2.11) can be solved. The solution for the velocity and induced magnetic field are

$$F(\eta) \frac{I}{m_1^2} \left[ \left\{ 1 - \frac{\cosh m_1 \eta}{\cosh m_1} \right\} + (G + G^*) \left\{ \frac{\sinh m_1 \eta}{\sinh m_1} - \eta \right\} \right] \tag{2.12}$$

$$h(\eta) \frac{(m_1^2 - 2iK^2)}{m_1^2 M^2} \left[ \frac{I}{m_1} \left\{ \frac{\sinh m_1 \eta}{\sinh m_1} + \frac{(G + G^*) I - \cosh m_1 \eta}{\sinh m_1} \right\} - \eta \left\{ \frac{G + G^*}{2} \eta \right\} \right] \tag{2.13}$$

Where  $m_1 = \alpha - i\beta$ ,

$$\alpha = \frac{I}{\sqrt{2}} \left[ \left\{ \frac{M^4}{I + m^2} + \frac{4M}{I + m^2} M^2 K^2 + 4K^4 \right\}^{1/2} + \frac{M^4}{I + m^2} \right]^{1/2}, \tag{2.14a}$$

$$\beta = \frac{I}{\sqrt{2}} \left[ \left\{ \frac{M^4}{I + m^2} + \frac{4M}{I + m^2} M^2 K^2 + 4K^4 \right\}^{1/2} - \frac{M^4}{I + m^2} \right]^{1/2} \tag{2.14b}$$

Shear stress at the plates  $\eta = \pm 1$

The non-dimensional shear stress components  $\tau_x$  and  $\tau_z$  at the plate  $\eta = \pm 1$  due to primary and secondary flows, respectively, are

$$\tau_x = \frac{1}{(\alpha^2 - \beta^2)} \left[ \mp \left( \frac{\alpha \sinh 2\alpha + \beta \sin 2\beta}{\cosh 2\alpha + \cos 2\beta} \right) + \left( G + G^* \frac{\beta \sinh 2\alpha + \alpha \sin 2\beta}{\cosh 2\alpha - \cos 2\beta} - \frac{2\alpha\beta}{\alpha^2 + \beta^2} \right) \right] \quad (2.16)$$

The upper and lower signs in the first term of eqns. (2.15) and (2.16) correspond to the values at the upper plate  $\eta = 1$  and that at the lower plate  $\eta = -1$  respectively.

It may be noted from (2.15) and (2.16) that the shear stress components  $\tau_x$  and  $\tau_z$  due to primary and secondary flows respectively, vanish neither at the upper plate nor at the lower plate and depend on the Hartmann number  $M$ , rotation parameter  $K^2$  and Hall parameter  $m$  when  $G = 0$  and  $G^* = 0$ . Thus it concludes and for perfectly conducting plates there is no flow reversal when  $G = 0$  and  $G^* = 0$

### Asymptotic Solutions:

**Case I :**  $K^2 \gg 1$  and  $M^2 \rightarrow 0$  (1) – Since  $A^2$  is very large and  $M^2$  and  $m$  are small orders of magnitude, it can expect boundary layer type flow. For the boundary layer at the upper plate  $\eta = 1$ , writing  $(1 - \eta) = \xi$ , we obtain from (2.12) and (2.13).

$$u_1 = \frac{e^{-\alpha\xi}}{2\lambda^2} (1 - G - G^*) \sin \beta\xi \quad (2.17)$$

$$w_1 = \frac{1}{2K^2} \left[ (1 - G\eta) + e^{\alpha\xi} (G + G^* - 1) \cos \beta\xi \right] \quad (2.18)$$

$$H_x^0 = \frac{1}{M^2} \left[ \frac{1}{K} \left\{ e^{\alpha\xi} (1 + G + G^*) (\cos \beta\xi - \sin \beta\xi) \right\} - \eta \left( 1 - \frac{G\eta}{2} - \frac{G^*\eta}{2} \right) \left( 2 + \frac{1 - m}{2(1 + m^2)K^2} \right) \right] \quad (2.19)$$

$$H_z = \frac{e^{-\alpha\xi}}{M^2 K} (1 + G + G^*) (\sin \beta\xi + \cos \beta\xi), \quad (2.20)$$

Where

$$\alpha = K \left\{ 1 + \frac{(1+M)M^2}{4(1+m^2)K^2} \right\}, \quad \beta = K \left\{ 1 + \frac{(1-M)M^2}{4(1+m^2)K^2} \right\} \quad (2.21)$$

It is evident from the expressions (2.17) to (2.20) that there arise boundary layer of thickness  $O(\alpha)^{-1}$  which decreases with the increase in either  $K^2$  and  $M^2$ . This boundary layer may be identified as modified hydromagnetic Ekman layer.

The exponential terms in eqns. (2.17) to (2.20) damp out quickly on  $\xi$  increases. When  $\xi \geq \frac{1}{\alpha}$ , we have

$$u_1'' = 0, w_1 = \frac{(1 - G\eta - G^*\eta)}{2K^2} \quad (2.22)$$

$$H_z = \frac{\eta}{M^2} \left( \frac{1 - G\eta}{2} - \frac{G^*\eta}{2} \right) \left( 2 + \frac{1 - M}{2(1+m^2)K^2} \right), H_x = 0 \quad (2.23)$$

The expressions (2.22) and (2.23) show that in a certain core, given by  $\xi \geq \frac{1}{\alpha}$  about the axis of the channel, the velocity in the direction of pressure gradient given by  $u_1$  and the induced magnetic field  $H_x$  vanish away while the velocity in the secondary flow direction given by  $w_1$  and the induced magnetic field  $H_x$  persist. Also the velocity  $w_1$  is unaffected by the Hall current and magnetic field. The velocity  $w_1$  and primary induced magnetic field  $H_x$  vary linearly with  $\eta$  and the effect of Grashof number on the velocity and induced magnetic field become insignificant in the central region.

**Case II :**  $M^2 \gg 1$  and  $K^2 \sim O(1)$  – In this case also boundary layer type flow is expected. For the boundary layer at the upper plate we obtain from (2.12) and (2.13)

$$u_1 = \frac{1}{M^2} [(1 + G^*\eta) + (+G^* - 1) e^{-\alpha\xi} (\cos \beta\xi + m \sin \beta\xi)], \quad (2.24)$$

$$w_1 = \frac{1}{M^2} [-M(1 - G + G^*\eta) + (+G^* - 1) e^{-\alpha\xi} (\sin \beta\xi - m \cos \beta\xi)], \quad (2.25)$$

$$H_x = \frac{1}{M^2} \left[ \frac{(1 + G + G^*)}{M^2} e^{-\alpha\xi} (\alpha \cos \beta\xi - \beta \sin \beta\xi) - \eta \left( 1 - \frac{G + G^*\eta}{2} \right) \right] \quad (2.26)$$

$$H_x = \frac{(1 + G + G^*)}{M^4} e^{-\alpha\xi} (\alpha \sin \beta\xi + \beta \cos \beta\xi) \quad (2.27)$$

Where

$$\alpha = \frac{M}{\sqrt{1+m^2}}, \beta = \frac{mM}{2\sqrt{1+m^2}} \quad (2.28)$$

The expressions (2.24) to (2.27) demonstrate the existence of a boundary layer of thickness  $O(\alpha)^{-1}$  which depends on both the Hall current and magnetic field. The thickness of this layer decreases with the increase in  $M^2$  while it increases with the increase in Hall parameter  $m$ . This boundary layer may be identified as modified Hartmann boundary layer. In the central core given by  $\xi \geq \frac{1}{\alpha}$  about the axis of the channel, the velocity field and magnetic field become

$$u_1 = \frac{1 - G + G^*\eta}{M^2}, w_1 = \frac{-m + (1 - G + G^*\eta)}{M^2} \quad (2.29)$$

$$H_x = -\eta \left( \frac{1 - \frac{G + G^*\eta}{2}}{M^2} \right), H_z = 0 \quad (2.30)$$

It is evident from the expressions (2.29) and (2.30) that in the central region the secondary velocity is weak in comparison to the primary velocity. In the absence of Hall current the secondary velocity  $w_1$  vanishes away and the fluid will be moving in the direction of pressure gradient only. Also both the velocity and the induced magnetic field  $H_x$  vary linearly with  $\eta$  and the effect of Grashof number on the velocity and induced magnetic field become insignificant.

### Heat transfer Characteristics

The energy equation for the fully-developed flow including viscous and Joule dissipations, reduces to

$$u \frac{(T - T_0)}{\partial x} = \frac{K}{\rho C_p} \frac{d^2(T - T_0)}{dy^2} + \frac{\mu}{\rho C_p} \left\{ \left( \frac{du'}{dy} \right)^2 + \left( \frac{dw'}{dy} \right)^2 \right\}$$



$$+ \frac{1}{\rho C_p \delta} \left\{ \left( \frac{dH'_x}{dy} \right)^2 + \left( \frac{dH'_z}{dy} \right)^2 \right\} \quad (2.31)$$

Where the fluid temperature T is a function of y only and other symbols have their usual meanings.

Using non-dimensional variable (2.8) and introducing dimensionless quantities

$$\theta(\eta) \frac{\phi(y)}{NLP_x}, \quad K_1 = \frac{v^3 P_x}{KNL^3} \quad \text{and} \quad P_r = \frac{\mu C_p}{K}, \quad (2.32)$$

in eqn. (2.31) we obtain

$$\frac{d^2\theta}{d\eta^2} = -K_1 \left[ \left\{ \left( \frac{du_1}{d\eta} \right)^2 + \left( \frac{dw_1}{d\eta} \right)^2 \right\} + M^2 \left\{ \left( \frac{dH'_x}{d\eta} \right)^2 + \left( \frac{dH'_z}{d\eta} \right)^2 \right\} \right] + P_r u_1 \quad (2.33)$$

The boundary conditions become

$$\theta(-1) = 0 \quad \text{and} \quad \theta(1) = \frac{\phi(1)}{NLP_x} = N_1 \quad (\text{say}), \quad (2.34)$$

Where  $N_1$  is the temperature at the upper plate. Substituting the values of  $u_1$ ,  $w_1$ ,  $H$ , and  $H_x$  from (2.12) and (2.13) in eqn. (2.33) and solving the resulting differential equation subject to the boundary conditions (2.34), the solution for  $\theta(\eta)$  may be represented as

$$\begin{aligned} \theta(\eta) = & P_r [\phi_1(\eta) + \phi_2(\eta) - A_1 \eta - A_4] - K_1 \{ \phi_3(\eta) + \phi_4(\eta) \\ & + \phi_5(\eta) + \phi_6(\eta) + A_2 \eta - A_3 \eta - A_5 - A_6 - A_7 \} + \frac{N_1}{2} (1 + \eta) + \phi_7(\eta) \end{aligned} \quad (2.35)$$

Where  $\phi(\eta)$  ( $i = 1, 2, \dots, 7$ ) are functions of  $M$ ,  $K^2$ ,  $m$ ,  $P_r$  and  $\eta$ .  $A_i$  ( $i = 1, 2, \dots, 7$ ) are function of  $M$ ,  $K^2$  and  $m$ .

The expression of rate of heat transfer at both the plates i.e.  $\left( \frac{d\theta}{d\eta} \right)_{\eta = \pm 1}$  are also derived. We omit these expressions because of quite lengthy.

**Mass transfer :**

concentration equation is given by

$$u' \frac{dc'}{dx'} = D \frac{d^2c'}{dy'^2}, \tag{2.36}$$

Introducing the following non-dimensional quantities

$$S_c = \frac{\nu}{D}, C = \frac{\phi'}{N'LP'_x}, \tag{2.37}$$

in the above equation, we get

$$\frac{d^2C}{d\eta^2} + RS_c \frac{dC}{d\eta} = 0, \tag{2.38}$$

with the boundary conditions (transferred)

$$C = 1 \text{ at } \eta = 0 \text{ \& } C = 0 \text{ at } \eta = 1$$

Solving the above equation,

$$\text{We get } C = \frac{1 - e^{RS_c(1-\eta)}}{1 - e^{RS_c}}, \tag{2.39}$$

Where R is the Reynolds number.

With the boundary conditions,

$$C' = 0 \text{ at } y' = 0$$

$$C' = C_0 \text{ at } y' = L$$

**3. DISCUSSION OF RESULTS**

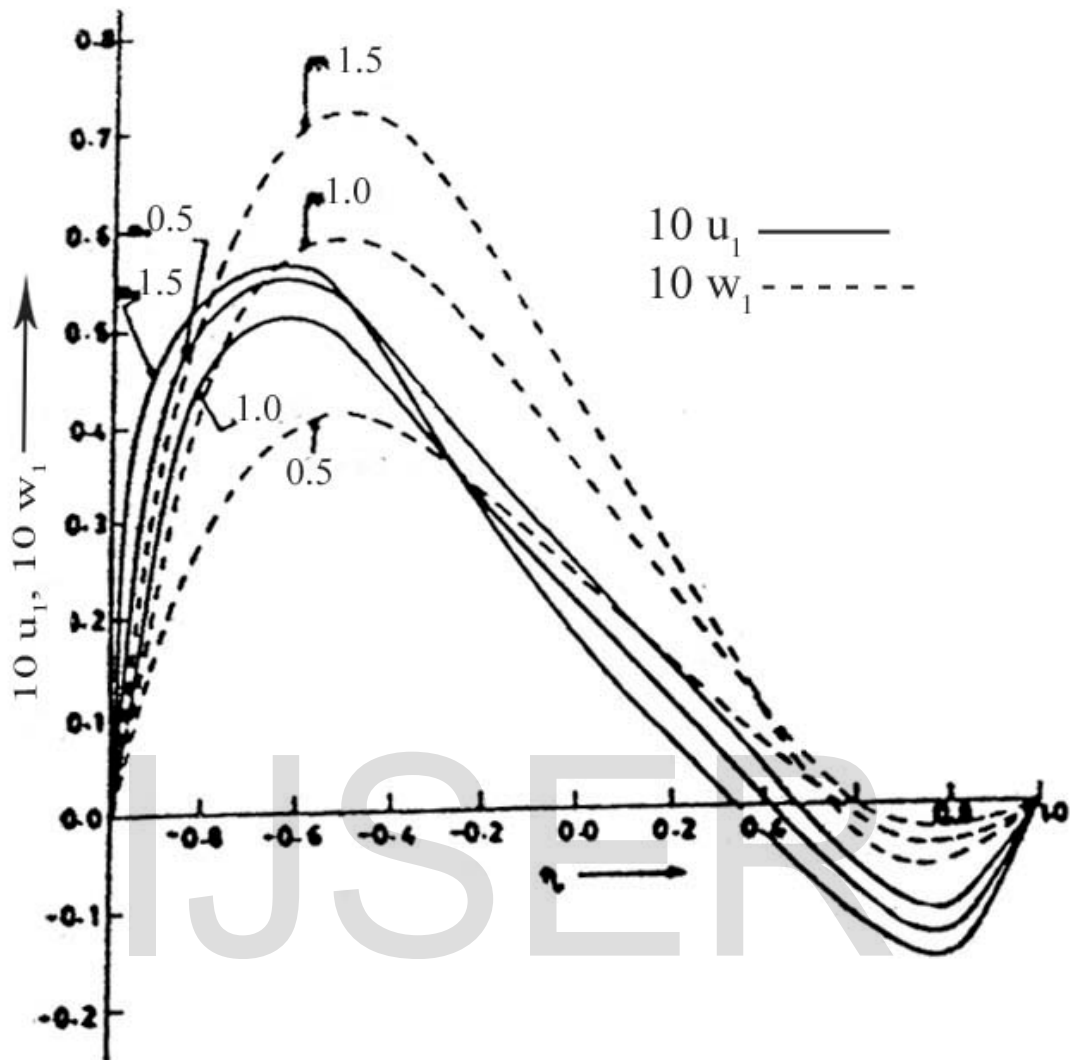
The numerical solutions of the velocity and the induced magnetic field are presented graphically versus  $\eta$  for various values of  $m^2$  taking G,  $K^2$  and M fixed in Fig.1 and 2. It is evident from Fig. 1 that for  $G > 0$ , the primary and secondary velocities change its direction as it move away from the upper half to the lower half of the channel and the velocity attains its maximum near the lower plate of the channel. Thus the free convection causes flow reversal in both the direction. It is observed from Fig. 1 that the primary velocity  $u_1$  is of oscillatory nature in the region – 1

$\leq \eta \leq 0.25$  as  $m$  increases whereas it decreases with the increase in  $m$  for  $0.35 \leq \eta \leq 1$  and the secondary velocity  $w_1$  increase in the region  $-1 \leq \eta \leq 0.4$  with the increase in  $m$  and as changes its direction near  $\eta=0.6$ , again increases with the increase in  $m$ . In this case there arise a flow reversal in the primary flow direction when  $\eta \geq 0.35$  while for secondary flow it appears  $\eta \geq 0.55$ . Fig. 2 reveals that the induced magnetic field  $H_x$  increases numerically with the increase in  $m$  while the induced magnetic field  $H_z$  increases numerically near the lower plate and decreases in magnitude in the region  $-0.5 < \eta < 0.7$  and again it increases in magnitude near the upper plate.

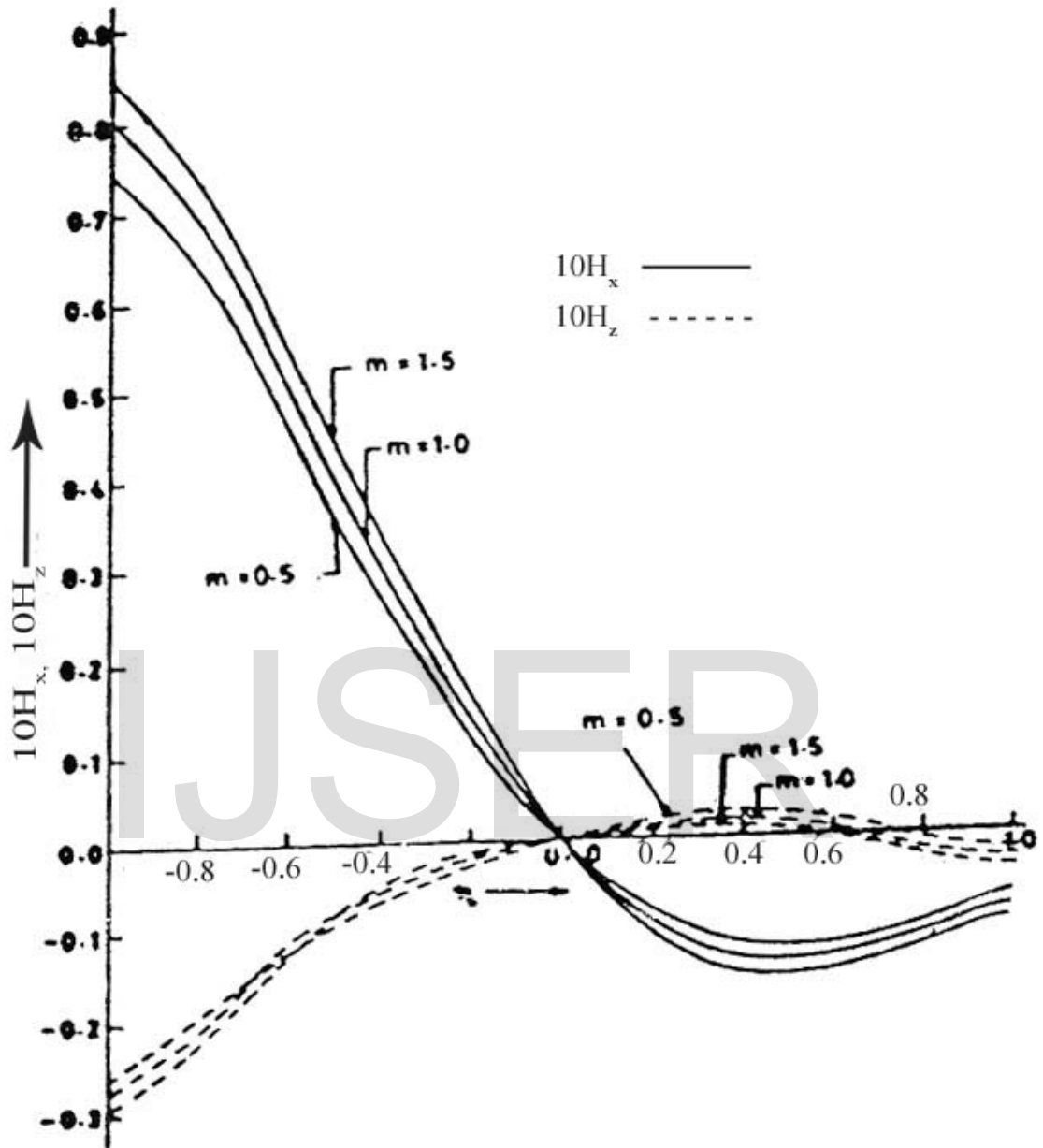
Fig.3 shows the velocity components  $u_1$  and  $w_1$  for various values of Grashof number ( $G$ ), Hartmann number ( $M$ ) and Rotation parameter ( $K^2$ ). It is noticed that both  $u_1$  and  $w_1$  first rises and then falls with the distance  $\eta$  from the lower plate to the upper plate of the channel. Further the increase in the value of  $G$  increases the velocity components  $u_1$  and  $w_1$  represented by the solid line and dotted line curves of the graph. When the external magnetic field strength is stronger, there is decrease in the value of  $u_1$  and  $w_1$ . Similar effect is marked in case of rotation parameter ( $K^2$ ) that decelerates the flow producing churning of the liquid.

Fig. 4 illustrates the fluid temperature for different values of Prandtl number ( $P_r$ ). It is seen that the increase in  $P_r$  reduces the temperature ( $\theta$ ).

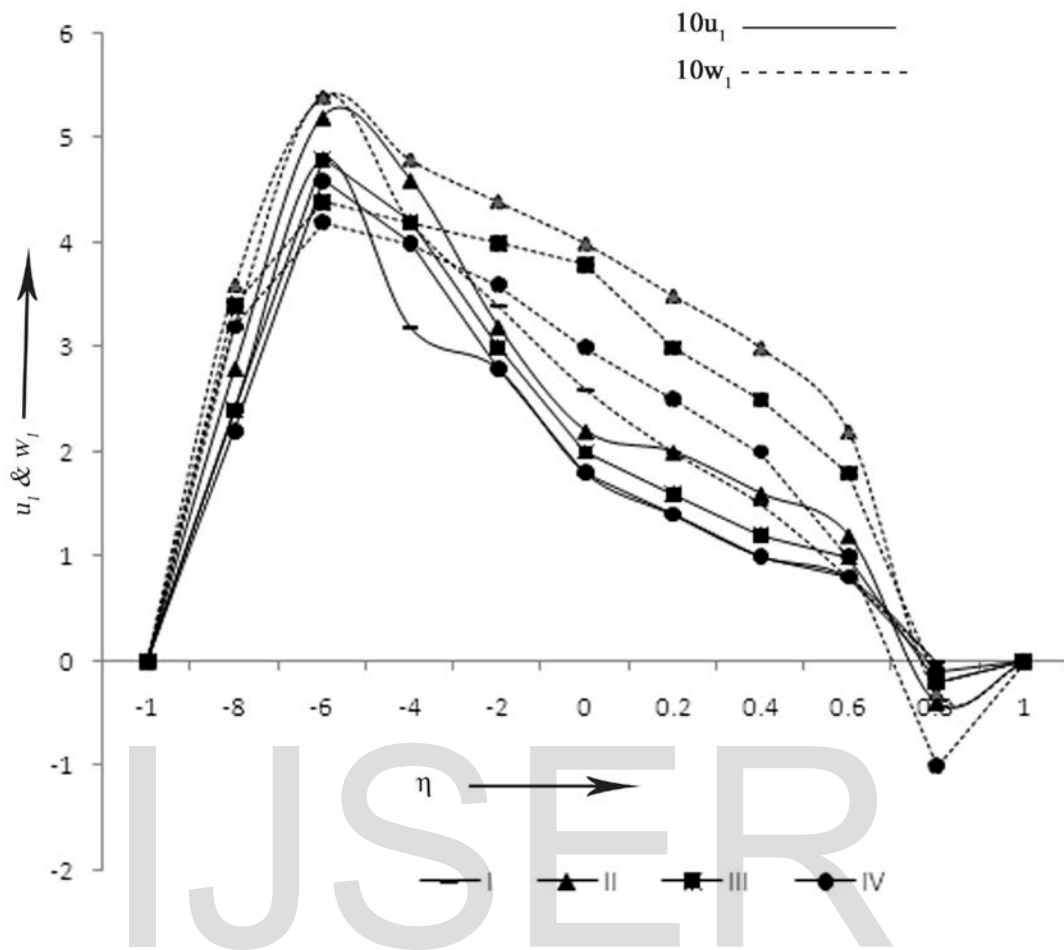
Fig. 5 explains the behaviour of the flow in respect of concentration. It is observed that the rise in the Schmidt number reduces the concentration.



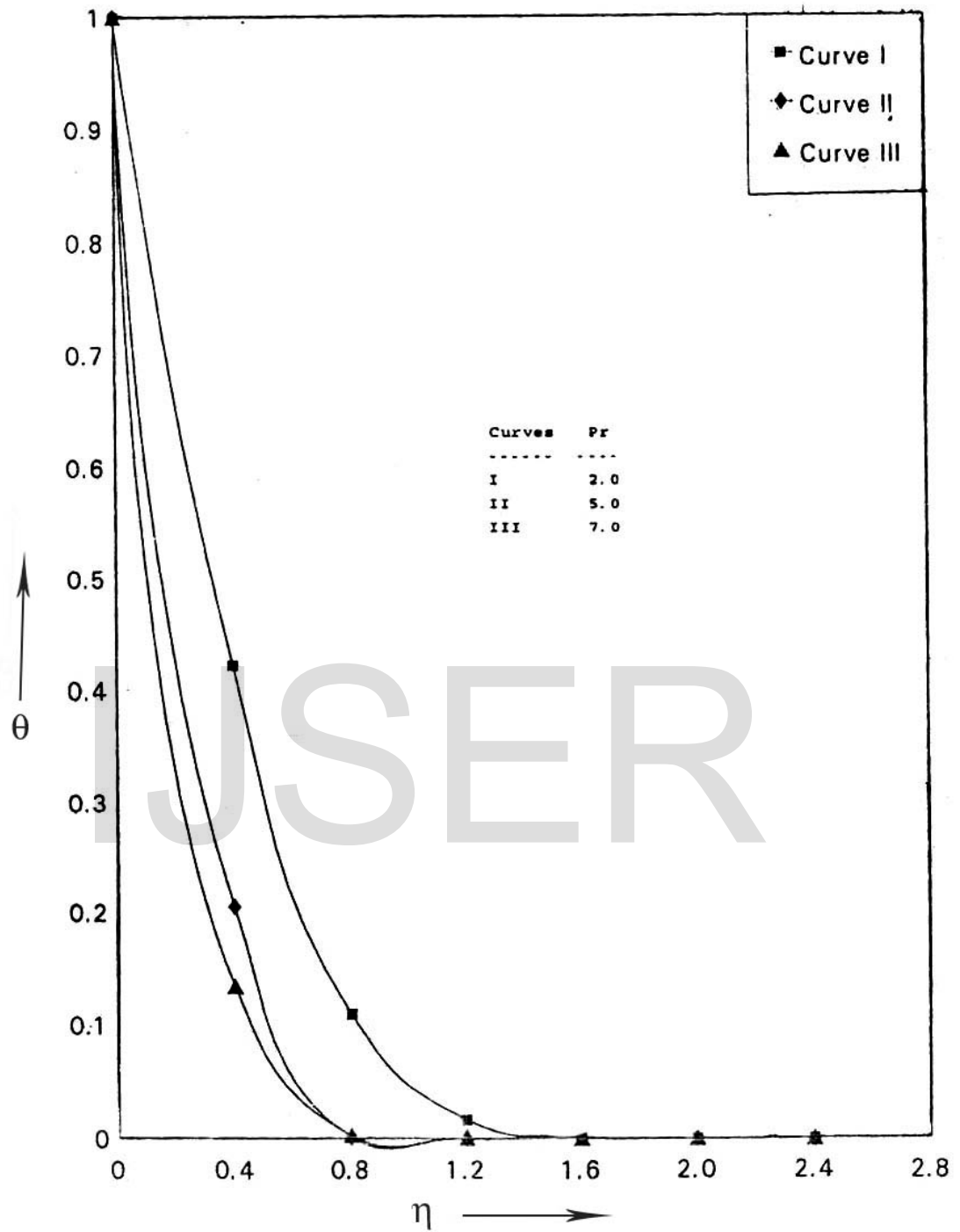
**Fig. 1 :** Effects of Hall parameter on the velocity components  $u_1$  and  $w_1$  for  $G = 2$ ,  
 $M = 5$ ,  $K^2 = 5$ .



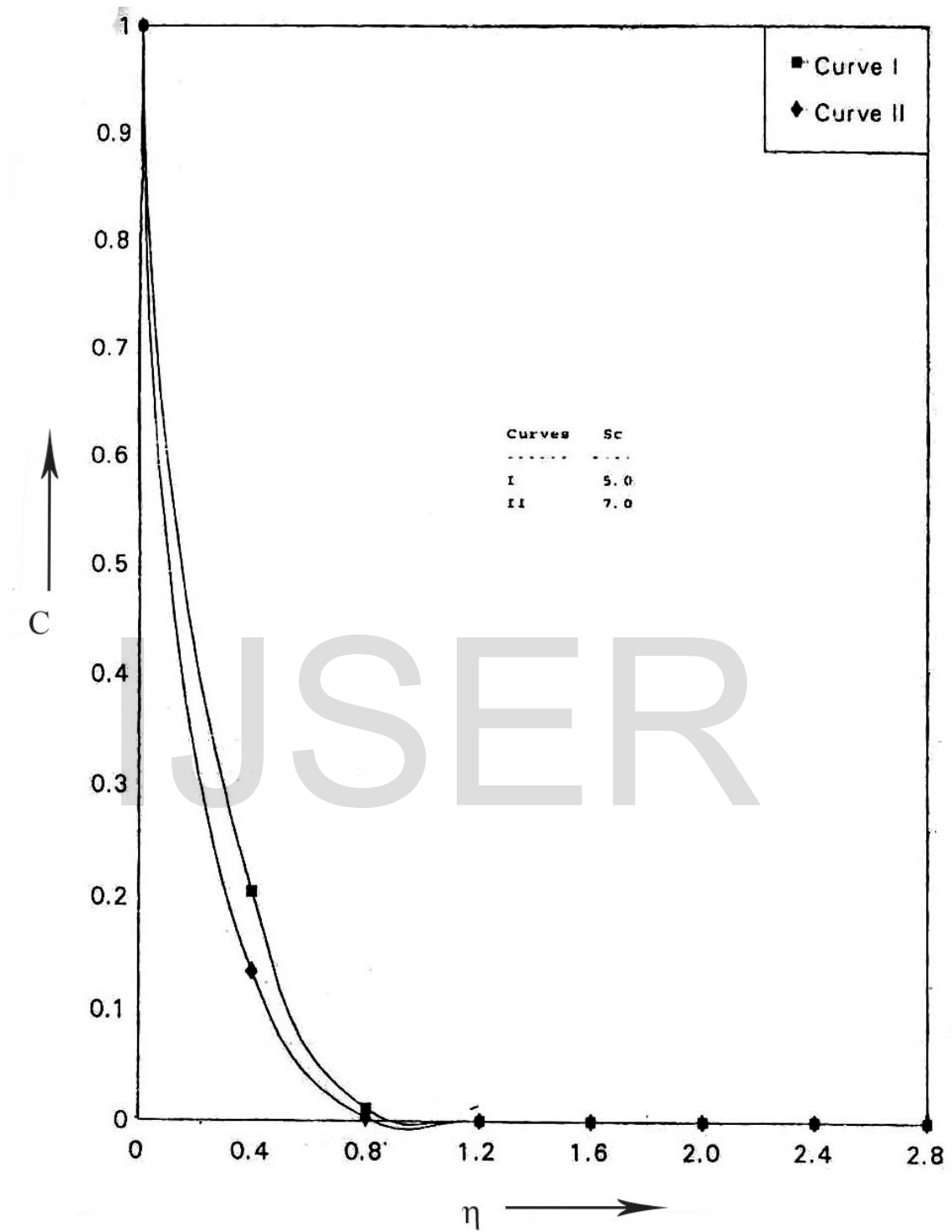
**Fig. 2 :** Effects of Hall parameter on magnetic field components for  $G = 2$ ,  $M = 5$ ,  $K^2 = 5$ .



**Fig. 3 :** Effects of  $G, M$  and  $K^2$  on the velocity components  $u_1$  and  $w_1$  for  $m=1.0$



**Fig. 4 :** Effects of Prandtl number  $P_r$  on temperature for  $m = 1.0$ ,  $M=2.0$  and  $K^2 = 5.0$



**Fig. 5 :** Effects of Schmidt number  $S_c$  on concentration for  $m=1.0$ ,  $M=2.0$ ,  $K^2=5.0$  and  $R=2.0$



**Table – I**

Shear stress  $\tau_x$  and  $\tau_z$  at  $\eta = 1$  for  $M = 5$  and  $K^2 = 3$

<b>G</b>	<b><math>\tau_x</math></b>			<b><math>\tau_z</math></b>		
	<b><math>m^2</math></b>	<b>0.5</b>	<b>1.0</b>	<b>1.5</b>	<b>0.5</b>	<b>1.0</b>
0.0	-0.18648	-0.18688	-0.19111	-0.06542	-0.09938	-0.12522
2.0	0.12541	0.13681	0.14989	-0.11420	-0.17360	-0.22086
4.0	0.43729	0.46050	0.49091	-0.16298	-0.24782	-0.31649
6.0	0.74918	0.78419	0.83192	-0.21176	-0.32205	-0.41214

**Table – II**

Shear stress  $\tau_x$  and  $\tau_z$  at  $\eta = -1$  for  $M = 5$  and  $K^2 = 3$

<b>G</b>	<b><math>\tau_x</math></b>			<b><math>\tau_z</math></b>		
	<b><math>m^2</math></b>	<b>0.5</b>	<b>1.0</b>	<b>1.5</b>	<b>0.5</b>	<b>1.0</b>
0.0	-0.18648	-0.18688	-0.19111	-0.06542	-0.09938	-0.12522
2.0	0.49836	0.51057	0.53212	0.01663	0.02515	0.02958
4.0	0.81025	0.83426	0.87314	-0.03217	-0.04906	-0.06605
6.0	1.12214	1.15796	1.21415	-0.08092	-0.12529	-0.16169

The numerical results of shear stress components  $\tau_x$  and  $\tau_z$  at both the plates due to primary and secondary flows, respectively are presented in Tables I and II for various values of G,  $m^2$  taking  $M = 5$  and  $K^2 = 3$ . Table I shows that the shear stress component  $\tau_x$  at  $\eta=1$  due to primary flow increases with the increase in either  $m^2$  or G while the shear  $\tau_z$  at  $\eta=1$  increases in either  $m^2$  or G while the shear stress component  $\tau_z$  at  $\eta = 1$  increases in magnitude with the increase in either  $m^2$  or G. Table II shows that the shear stress component  $\tau_x$  at  $\eta = -1$  due to primary flow increases with the increase in either  $m^2$  or G while the shear stress component  $\tau_z$  at  $\eta$

= -1 increases with the increase in G for fixed  $m^2$  whereas it decreases with the increase in  $m^2$  for fixed value of G. Also it is noticed that there arise separation of flow on increasing G for fixed value of  $m^2$ .

**Rate of heat transfer :**

The rate of heat transfer at both the plates i.e.  $\left(\frac{d\theta}{d\eta}\right)_{\eta=\pm 1}$  are presented in Table III, for various values of m and G taking  $M = 5$ ,  $K^2 = 5$ ,  $N_1 = 1$  and  $P_r = 0.25$ . It is observed from Table III that the rate of heat transfer  $\left(\frac{d\theta}{d\eta}\right)_{\eta=1}$  decreases with the increase in either m of G whereas the rate of heat transfer  $\left(\frac{d\theta}{d\eta}\right)_{\eta=-1}$  increases with the increase in G. Also it increases with the increase in m for  $G = 0$  (forced convection) and is of oscillatory nature on increasing m for  $G \neq 0$ . It is noticed that there arise reverse flow of heat at the upper plate on increasing m for  $G=2$ . Thus we conclude that the rotation, Hall current and heat transfer by free convection induce reverse flow of heat at the upper plate.

**Table – III**

The rate of heat transfer  $\left(\frac{d\theta}{d\eta}\right)_{\eta=1}$  for  $M = 5$  and  $K^2 = 5$

G	$\left(\frac{d\theta}{d\eta}\right)_{\eta=1}$				$\left(\frac{d\theta}{d\eta}\right)_{\eta=-1}$			
	0	0.5	1.0	1.5	0	0.5	1.0	1.5
0	0.16203	0.14530	.012817	.11252	.33797	.35469	-.7183	.38747
2	0.06223	0.00335	-.001608	-.04228	-0.56297	.58753	-52640	-54701
4	-.22873	-.39892	-.45460	-.53436	-97919	1.08069	.97523	1.04382
10	-1.02249	-3.16775	-3.53574	-4.03421	3.37513	4.12218	4.08732	4.55787

**Concentration gradient :**

The concentration gradient is given by  $\left(\frac{dC}{d\eta}\right)_{\eta=\pm l}$  for  $M = 5, K^2 = 5, m = 1.0$

$$CG_1 = \left.\frac{dc}{d\eta}\right|_{\eta=l} \qquad CG_2 = \left.\frac{dc}{d\eta}\right|_{\eta=-l}$$

**Table – IV**

<b>R</b> \ <b>S<sub>c</sub></b>	<b>0.81</b>		<b>1.5</b>		<b>2.7</b>	
	<b>CG<sub>1</sub></b>	<b>CG<sub>2</sub></b>	<b>CG<sub>1</sub></b>	<b>CG<sub>2</sub></b>	<b>CG<sub>1</sub></b>	<b>CG<sub>2</sub></b>
1.0	0.2203	0.2024	0.2392	0.2354	0.2477	0.2468
2.0	0.3700	0.3251	0.4340	0.4120	0.4737	0.4457
3.0	0.7830	0.4542	0.5907	0.5602	0.6733	0.6225
4.0	0.5762	0.5381	0.7221	0.6125	0.8489	0.7239
5.0	0.6571	0.5270	0.8364	0.7138	1.0052	0.9856

Table – IV presents the values of concentration gradient  $CG_1$  and  $CG_2$  for various values of Reynolds number (R) and Schmidt number ( $S_c$ ). It is observed that the increase of  $S_c$  increases both  $CG_1$  and  $CG_2$ . Similar result is obtained in case of rise of R.

### Conclusion:

Theoretical study of Hall effects on hydromagnetic non-Newtonian convective flow in a rotating channel with mass transfer reveals the following results.

- i) The primary velocity  $u_1$  is of oscillatory nature in the region  $-1 \leq \eta \leq 1$ .
- ii) The secondary velocity  $w_1$  increases in the region  $-1 \leq \eta \leq 0.4$  with the increase in Hall parameter.
- iii) The induced magnetic field  $H_x$  increases numerically with the increase in  $m$ .
- iv) Temperature decreases with the increase of Prandtl number  $P_r$ .
- v) Concentration decreases with the increase of Schmidt number  $S_c$ .
- vi) The shear stress component  $\tau_x$  at  $\eta = 1$  due to primary flow increases with the increase in either  $m^2$  or  $G$  while the shear stress component  $\tau_z$  at  $\eta=1$  decreases in magnitude with the increase in either  $m^2$  or  $G$ .
- vii) The rate of heat transfer  $Nu_1$  decreases with the increase in either  $m$  or  $G$  whereas the rate of heat transfer  $Nu_2$  increases with the increase in  $G$ .
- viii) Increase in the Schmidt number  $S_c$  increases both the concentration gradient  $CG_1$  and  $CG_2$  at the upper and lower plate of the channel.

## REFERENCES :

1. B.S. Mazumder, A.S. Gupta and N. Datta, Int. J. Engng. Sci. 14 (1976), 285.
2. N. Datta and R.N. Jena, Int. J. Engng. Sci. 15 (1977), 561.
3. G.S. Seth and S.K. Ghosh, Proc. Math. Soc. 3 (1987)
4. J. Hartmann, Kgl. Danske. Viden Selsk (Math. Fys. Meddele), Vol 15 No. 6 (1937), 1-50.
5. J.P. Agrawal, appl. Sci. Res., 9B (1962), 255.
6. V.M. Soundalgekar Proc. Nat. Sci Ind. 35 (1969) 329
7. S.K. Ghosh, Ind Journal of pure and applied Math 25 (9), (1994), 991.
8. S.S.S. Mishra, S. Biswal, G.S. Roy, app. Sci. Periodical 14 ((2012)
9. M. Jena, M. Goswami and S. Biswal, proc. Nat. Sci. Ind (Communicated), 2012.
10. S. Biswal, G.S. Ray & P.K. Mishra Ultra Scientist, Vol. 23 (1), (2011) 189-204
11. R.S. Nanda and H.K. Mohanty, Appl. Sci. Res. 24 (1971), 65.